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One-loop Singular Behaviour of QCD and SUSY QCD Amplitudes with Massive Partons

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Abstract

We discuss the structure of infrared and ultraviolet singularities in on-shell QCD and supersymmetric QCD amplitudes at one-loop order. Previous results, valid for massless partons, are extended to the case of massive partons. Using dimensional regularization, we present a general factorization formula that controls both the singular ϵ-poles and the logarithmic contributions that become singular for vanishing masses. We introduce generalized Altarelli–Parisi splitting functions and discuss their relations with the singular terms in the amplitudes. The dependence on the regularization scheme is also considered.

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1 Introduction

Hard-scattering processes that lead to multiparton final states with heavy particles are important for present physics studies within and beyond the Standard Model. Their importance will further increase at future high-energy colliders (see, e.g., Refs. [1, 2] and references therein). Reliable theoretical predictions for these processes require the evaluation of (at least) next-to-leading order QCD corrections, independently of the nature (QCD, electroweak, SUSY) of the interaction that produces these processes at the lowest perturbative order. The computation of QCD radiative corrections to cross sections that involve heavy partons are certainly more complicated than in the case of massless partons, as known since the first NLO calculations of heavy-quark production in hadron collisions [3].

Most of the available techniques (see, e.g., the list of references in Sect. 4 of the QCD Chapter of Ref. [2]) to perform NLO calculations require the analytic evaluation of the infrared (soft and collinear) and ultraviolet singularities of the one-loop amplitudes for the corresponding process. The knowledge of the singularity structure in a general (process-independent) form is thus useful for several reasons. Loop calculations are always very cumbersome and error-prone, therefore, knowing any general property of the result is a good check on the calculations. The knowledge of the singularities of the loop amplitude can also be used as a possible tool to split the loop calculation into a divergent part, which is known in analytic form, and a finite part, to be evaluated by numerical methods. In the case of infrared-safe observables the infrared singularities of the real and loop corrections must cancel, therefore, knowing the singularity structure without performing the explicit loop computation helps devising process-independent calculational schemes at NLO.

The singular behaviour of QCD amplitudes involving massless partons is completely known at one-loop order [4–6], and the coefficients of the divergent $1/\epsilon^n$ poles for n =2, 3 and 4 are also known for two-loop amplitudes in the most general case [7]. Despite the numerous one-loop calculations involving massive particles that have been performed so far, to our knowledge the general form of the singularities in amplitudes with massive partons has not yet been presented even at one-loop order.

In this paper we discuss the structure of the singularities of one-loop QCD (or QED) corrections to on-shell scattering amplitudes that involve both massless and massive partons. By singularities we mean not only the terms that diverge in absence of regularization, but also those that become divergent in the limit of vanishing parton masses. With increasing collision energies the ratios of the particle masses to the kinematic (Mandelstam) invariants tend to zero. Thus the amplitude contains logarithms of such ratios that become large and whose explicit control is important for numerical stability. This feature has particular relevance for electroweak physics and one-loop QED corrections where collinear singularities are physically regularized by the small values of the light-fermion masses.

We present a universal (process-independent) factorization formula that embodies the structure and the coefficients of all these singular terms in the one-loop amplitudes. We use dimensional regularization in $d = 4 - 2\epsilon$ dimensions for both ultraviolet and infrared divergences, and we thus give the coefficients[‡] of the corresponding poles $1/\epsilon^2$ and $1/\epsilon$. The

[‡]These coefficients can also be obtained by using the general results of Ref. [8].

factorization formula also explicitly exhibits all logarithmically-enhanced terms $\ln^2 m$ and $\ln m$ that become singular for vanishing mass m of the massive partons. Furthermore, we can have full control of all the terms that are not analytic in the massless limit, because the factorization formula also contains the constant (when $m \to 0$) terms that originate from the non-commutativity of the limits $m \to 0$ and $\epsilon \to 0$. Therefore, in the limit $m \to 0$, the finite (when $\epsilon \to 0$) one-loop contribution that is not included in the factorization formula tends smoothly to the finite contribution of the corresponding massless amplitude.

There are two ways to derive the singular terms. They can be obtained either by explicitly performing loop calculations or by using unitarity. We use the latter method. It amounts to exploiting the fact that the infrared singularities of the real and loop corrections cancel each other in the computation of infrared-safe observables. Thus, the singular structure of the loop corrections can be derived from that of the real corrections. Our process-independent calculation of the real corrections is based on the dipole subtraction formalism [6]. In particular, we have explicitly extended the results of Refs. [6, 9] to the general case of partons with arbitrary masses. In this paper we present only the final results on the singular behavior of one-loop amplitudes. Complete details of the extended formalism will be presented elsewhere.

After fixing our notation and conventions in Sect. 2, we present our results on the singular terms of QCD and SUSY QCD amplitudes in Sect. 3. The singularities produced by QED corrections to electroweak and QCD amplitudes can be obtained from the QCD results as a special case. In Sect. 4 we relate the singularities to individual terms in generalized Altarelli–Parisi functions. Section 5 contains our summary, and the Appendix provides a list of generalized Altarelli–Parisi functions.

2 Notation

2.1 Dimensional regularization

In the evaluation of loop amplitudes one encounters ultraviolet and infrared divergences that have to be properly regularized. The most efficient method to simultaneously regularize both types of singularities in gauge theories is to use dimensional regularization [10]. The key ingredient of dimensional regularization is the analytic continuation of loop momenta to $d = 4 - 2\epsilon$ space-time dimensions. Having done this, one is left with some freedom regarding the dimensionality of the momenta of the external particles as well as the number of polarizations of both external and internal particles. This leads to different regularization schemes (RS) within the dimensional-regularization prescription (see, e.g., Refs. [11, 12]).

The two variants of dimensional regularization that are mostly used in one-loop computations are conventional dimensional regularization (CDR) and dimensional reduction (DR). In both schemes one considers 2 helicity states for spin-$\frac{1}{2}$ Dirac fermions. The essential difference between the two schemes regards the number of the helicity states of the gluons in the loop. The gluon has $d-2$ helicities in CDR and 2 helicities in DR. Since the number of helicity states of gluons and Dirac fermions is the same in DR, this scheme

preserves supersymmetric Ward identities. Within each scheme one can still choose the external particles (their momenta and helicities) in the amplitudes to be either d-dimensional or 4-dimensional. At one-loop order, these choices lead to differences of $\mathcal{O}(\epsilon)$, which do not have any effect on the results presented in this paper.

Note that ultraviolet and infrared divergences behave differently with respect to dimensional-regularization prescriptions. The ultraviolet RS dependence can ultimately be removed by a proper redefinition of the renormalized running coupling. The infrared RS dependence instead leads to contributions (which are not vanishing for $\epsilon \to 0$) that depend on the specific amplitude and that cannot be reabsorbed by an overall (i.e., independent of the amplitude) redefinition of the renormalized coupling. These features, which were explicitly pointed out in Ref. [11], will be discussed in detail in the following sections. Note also that, in the calculation of physical quantities, the RS dependence of loop amplitudes has to be consistently matched to that of tree amplitudes. This issue is discussed on quite a general basis in Ref. [12].

2.2 Partially renormalized amplitudes

We consider amplitudes $\mathcal{A}_m$ that involve m external coloured particles (gluons, massless and massive quarks, gluinos and squarks) with momenta $p_1, \ldots, p_m$, masses $m_1, \ldots, m_m$ and an arbitrary number and type of colourless particles (photons, leptons, vector bosons, etc.). We always consider the amplitudes in the crossing-symmetric, but unphysical channel when all particle momenta are outgoing. The amplitudes are denoted by $\mathcal{A}_m(p_1, m_1, \ldots, p_m, m_m)$ (or, shortly, $\mathcal{A}_m(\{p_i, m_i\})$), and the dependence on the momenta and quantum numbers of the colourless particles is always understood.

For amplitudes of processes involving massive particles the $\overline{\rm MS}$ subtraction scheme is not always used to perform charge (coupling) renormalization (see, e.g., Ref. [13]). To leave the freedom of choosing a favourite charge-renormalization scheme, we find it convenient to use *mass-renormalized*, but *charge-unrenormalized amplitudes*. Thus, in the perturbative expansion

$$\mathcal{A}_m(g_{\rm s}, \mu^2; \{p_i, m_i\}) = \left(\frac{g_{\rm s}\mu^\epsilon}{4\pi}\right)^q \left[\mathcal{A}_m^{(0)}(\{p_i, m_i\}) + \left(\frac{g_{\rm s}}{4\pi}\right)^2 \mathcal{A}_m^{(1)}(\mu^2; \{p_i, m_i\}) + {\rm O}(g_{\rm s}^4)\right] \quad (1)$$

$g_{\rm s}$ stands for the *bare* strong coupling and m_i are the *renormalized* mass parameters (i.e., masses and related parameters such as those which appear in Yukawa couplings). The renormalized masses are obtained from the bare masses $m_i^{(0)}$ by the replacement $m_i^{(0)} \to m_i = m_i^{(0)} + g_{\rm s}\, \delta m_i$, so that the (ultraviolet-divergent) mass renormalization constants δm_i are implicitly contained in $\mathcal{A}_m^{(1)}$. In Eq. (1) q is a non-negative integer, and μ is the dimensional-regularization scale.

Equation (1) fixes the normalization of the tree-level, $\mathcal{A}_m^{(0)}$, and one-loop, $\mathcal{A}_m^{(1)}$, coefficient amplitudes that we use in the rest of the paper[§]. Although it is not explicitly denoted in

[§]Precisely speaking, $\mathcal{A}_m^{(0)}$ is not necessarily a tree amplitude, but rather the lowest-order amplitude for a given process; $\mathcal{A}_m^{(1)}$ is the corresponding NLO correction. For instance, in the case of $gg \to \gamma\gamma$, $\mathcal{A}^{(0)}$ involves a quark loop.

Eq. (1), $\mathcal{A}^{(0)}$ and $\mathcal{A}^{(1)}$ are both dependent on the RS.

2.3 Colour space

We shall present the singular structure of the QCD and SUSY QCD amplitudes directly in colour space. In particular, we use the same notation as in Ref. [6].

The colour indices of the m partons in the amplitude $\mathcal{A}_m$ are generically denoted by $c_1, \ldots, c_m$: $c_i = \{a\} = 1, \ldots, N_c^2 - 1$ for particles in the adjoint representation (gluons, gluinos) and $c_i = \{\alpha\} = 1, \ldots, N_c$ for particles in the fundamental representation (quarks, squarks and their antiparticles) of the gauge group. We formally introduce an orthogonal basis of unit vectors $\{|c_1, \ldots, c_m\rangle\}$ in the m-parton colour space, in such a way that the colour amplitude can be written as follows:

$$\mathcal{A}_m^{c_1,\ldots,c_m}(p_1, m_1, \ldots, p_m, m_m) \equiv \langle c_1, \ldots, c_m \,|\, \mathcal{A}_m(p_1, m_1, \ldots, p_m, m_m)\rangle \;. \tag{2}$$

Thus $|\mathcal{A}_m(p_1, m_1, \ldots, p_m, m_m)\rangle$ is an abstract vector in colour space, and the square amplitude summed over colours is

$$|\mathcal{A}_m(\{p_i, m_i\})|^2 = \langle \mathcal{A}_m(\{p_i, m_i\}) \,|\, \mathcal{A}_m(\{p_i, m_i\})\rangle \;. \tag{3}$$

Colour interactions at the QCD vertices are represented by associating a colour charge $\mathbf{T}_i$ with the emission of a gluon from each parton i. The colour charge $\mathbf{T}_i = \{T_i^a\}$ is a vector with respect to the colour indices a of the emitted gluon and an $SU(N_c)$ matrix with respect to the colour indices of the parton i. More precisely, its action onto the colour space is defined by

$$\langle c_1, \ldots, c_i, \ldots, c_m \,|\, T_i^a \,|\, b_1, \ldots, b_i, \ldots, b_m\rangle = \delta_{c_1 b_1} \ldots T^a_{c_i b_i} \ldots \delta_{c_m b_m} \;, \tag{4}$$

where T^a_{cb} is the colour-charge matrix in the representation of the final-state emitting particle i, i.e. $T^a_{cb} = i f_{cab}$ if i is a gluon or a gluino, $T^a_{\alpha\beta} = t^a_{\alpha\beta}$ if i is a (s)quark, and $T^a_{\alpha\beta} = -t^a_{\beta\alpha}$ if i is an anti(s)quark. The colour-charge operator of an initial-state parton is defined by crossing symmetry, that is by $(\mathbf{T}_i)^a_{\alpha\beta} = -t^a_{\beta\alpha}$ if i is an initial-state (s)quark and $(\mathbf{T}_i)^a_{\alpha\beta} = t^a_{\alpha\beta}$ if i is an initial-state anti(s)quark.

In this notation, each vector $|\mathcal{A}_m(p_1, m_1, \ldots, p_m, m_m)\rangle$ is a colour singlet, so colour conservation is simply

$$\sum_{i=1}^{m} \boldsymbol{T}_i \,|\mathcal{A}_m\rangle = 0 \;. \tag{5}$$

The colour-charge algebra for the product $(\mathbf{T}_i)^a (\mathbf{T}_j)^a \equiv \mathbf{T}_i \cdot \mathbf{T}_j$ is:

$$\boldsymbol{T}_i \cdot \boldsymbol{T}_j = \boldsymbol{T}_j \cdot \boldsymbol{T}_i \quad \text{if} \;\; i \neq j; \qquad \boldsymbol{T}_i^2 = C_i \;, \tag{6}$$

where C_i is the quadratic Casimir operator in the representation of particle i, and we have $C_F = T_R(N_c^2 - 1)/N_c = (N_c^2 - 1)/(2N_c)$ in the fundamental and $C_A = 2\,T_R N_c = N_c$ in the adjoint representation, i.e. we are using the customary normalization $T_R = 1/2$.

We remind the reader that, in the cases of amplitudes with $m = 2$ and $m = 3$ coloured partons, the colour-charge algebra can always be recast in a fully factorized form in terms of Casimir operators of the m partons (see the Appendix A of the second paper in Ref. [6]).

Note that in the following sections we always refer to one-loop QCD corrections to scattering amplitudes. Nonetheless, most of the results can straighforwardly be used for the case of one-loop QED corrections. To this purpose it is sufficient to replace the colour couplings $g_s \boldsymbol{T}_i$ by the electric couplings $g e_i$, where g is the electric charge of the electron and e_i is the charge of the parton i in units of the electron charge. In terms of colour factors, this implies the replacements $C_F \to e_i^2$, $T_R \to 1$ and $C_A \to 0$.

3 Singular behaviour at one-loop order with massive partons

3.1 QCD amplitudes

In this subsection we present our results on the singular behaviour of QCD amplitudes at one-loop order. In the massless case the one-loop coefficient subamplitude $\mathcal{A}_m^{(1)}$ has double and single poles in ϵ that can be obtained by a process-independent factorization formula [4–6]. Similar poles, although with different coefficients, appear if the amplitude involves massive partons. We find that these singularities are still universal, so that the factorization formula for the massless amplitudes can be generalized to the massive case.

The general factorization formula for the one-loop coefficient subamplitude $\mathcal{A}_m^{(1)}$ is¶

$$|\mathcal{A}_m^{(1)}(\mu^2;\{p_i,m_i\})\rangle_{\rm R.S.} = \boldsymbol{I}_m^{\rm R.S.}(\epsilon,\mu^2;\{p_i,m_i\})\,|\mathcal{A}_m^{(0)}(\{p_i,m_i\})\rangle_{\rm R.S.} + |\mathcal{A}_m^{(1)\,{\rm fin}}(\mu^2;\{p_i,m_i\})\rangle + {\rm O}(\epsilon). \tag{7}$$

All the ϵ-poles are included in the factor $\boldsymbol{I}$, so the remaining contributions on the right-hand side can be safely expanded in ϵ for $\epsilon \to 0$. Moreover, the factor $\boldsymbol{I}$ includes also the constant (when $\epsilon \to 0$) terms related to the RS dependence. Thus, the contribution $\mathcal{A}_m^{(1)\,{\rm fin}}$ is not only finite, but it is also RS-independent.

These features are shared by massless and massive amplitudes. In the case of massive quarks, moreover, our factorization formula has additional important properties related to logarithmically-enhanced contributions. In the limit of one or more vanishing masses m_i, the subamplitude $\mathcal{A}_m^{(1)}(\mu^2;\{p_i,m_i\})$ contains *logarithmic* terms of the type $\ln^2 m_i$ and $\ln m_i$ that become singular. It also contains *constant* terms that originate from the non-commutativity of the limits $m_i \to 0$ and $\epsilon \to 0$. We are able to embody all these logarithmic and constant terms‖ in the factor $\boldsymbol{I}$. Thus, in the limit $m_i \to 0$ the finite contribution $\mathcal{A}_m^{(1)\,{\rm fin}}(\dots,p_i,m_i,\dots)$ tends smoothly to the finite contribution $\mathcal{A}_m^{(1)\,{\rm fin}}(\dots,p_i,m_i=0,\dots)$

¶Here and in the following, the labels R.S. explicitly denote the RS dependence of the various quantities.

‖In the small-mass limit, the mass m_i replaces dimensional regularization as regulator of collinear singularities. In this sense, we can say that we can control the ensuing regularization-scheme dependence including finite (when $m_i \to 0$) terms.

of the corresponding amplitude in the theory where the parton i is massless:

$$\lim_{m_i \to 0} \mathcal{A}_m^{(1)\,\mathrm{fin}}(\mu^2; p_1, m_1, \ldots, p_i, m_i, \ldots) = \mathcal{A}_m^{(1)\,\mathrm{fin}}(\mu^2; p_1, m_1, \ldots, p_i, m_i = 0, \ldots) \,. \tag{8}$$

Note that Eq. (8) is valid independent of the actual definition of the renormalized mass m_i (or of the related Yukawa couplings). We can use either the pole-mass definition or the $\overline{\mathrm{MS}}$ definition, because the terms $\ln m_i$ originating from the different definitions are always suppressed by the corresponding mass factor as $m_i \ln m_i$.

The factorization formula (7) and the property in Eq. (8) can be used to check the calculation of the massive amplitude by comparing it to the corresponding massless calculation. In the asymptotic regime where m_i is much smaller than any of the relevant kinematic invariants Q, these equations can also be used to directly obtain (apart from corrections of $\mathcal{O}(m_i/Q)$) the one-loop massive amplitude from the corresponding massless amplitude, without explicitly computing the former.

The first term on the right-hand side of Eq. (7) has a factorized structure in colour space. The singular dependence (poles in ϵ and logarithms in m_i) is embodied in the factor $\boldsymbol{I}_m^{\mathrm{R.S.}}$ that acts as a colour-charge operator onto the colour vector $|\mathcal{A}_m^{(0)}\rangle_{\mathrm{R.S.}}$. Note that both factors are RS-dependent. In particular, the product of the RS-dependent terms of $\mathrm{O}(\epsilon)$ in $\mathcal{A}_m^{(0)}$ and double poles $1/\epsilon^2$ in $\boldsymbol{I}$ produces, in general, an RS dependence of $\mathcal{A}_m^{(1)}$ that begins at $\mathrm{O}(1/\epsilon)$.

The explicit expression for $\boldsymbol{I}_m$ in terms of the colour charges of the m partons is the following:

$$\begin{aligned}
\boldsymbol{I}_m^{\mathrm{R.S.}}(\epsilon, \mu^2; \{p_i, m_i\}) = \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \Bigg\{ & q\, \frac{1}{2} \left(\frac{\beta_0}{\epsilon} - \tilde{\beta}_0^{\mathrm{R.S.}} \right) \\
& + \sum_{\substack{j,k=1 \\ k \neq j}}^{m} \boldsymbol{T}_j \cdot \boldsymbol{T}_k \left(\frac{\mu^2}{|s_{jk}|} \right)^{\epsilon} \left[\mathcal{V}_{jk}^{(\mathrm{cc})}(s_{jk}; m_j, m_k; \epsilon) + \frac{1}{v_{jk}} \left(\frac{1}{\epsilon}\, \mathrm{i}\pi - \frac{\pi^2}{2} \right) \Theta(s_{jk}) \right] \\
& - \sum_{j=1}^{m} \Gamma_j^{\mathrm{R.S.}}(\mu, m_j; \epsilon) \Bigg\} \,.
\end{aligned} \tag{9}$$

The first term on the right-hand side of Eq. (9) contains the ultraviolet divergences, to be removed by the renormalization of the bare coupling g_{s}. It is proportional to q, which is the overall power of g_{s} in Eq. (1), and β_0 is the first coefficient of the QCD beta function

$$\beta_0 = \frac{11}{3}\, C_A - \frac{4}{3}\, T_R (N_f + N_F) \,, \tag{10}$$

where N_f and N_F are the numbers of *massless* and *massive* quark flavours, respectively. The constant coefficient $\tilde{\beta}_0^{\mathrm{R.S.}}$ parametrizes the ultraviolet RS dependence. Setting $\tilde{\beta}_0^{\mathrm{CDR}} = 0$ by definition in the CDR scheme, its corresponding value in the DR scheme is [14] (see also Ref. [11])

$$\tilde{\beta}_0^{\mathrm{DR}} = \frac{1}{3}\, C_A \,. \tag{11}$$

The second and third terms on the right-hand side of Eq. (9) have an infrared origin. We have determined them by exploiting the fact that, in NLO calculations of infrared-safe cross sections, the infrared singularities of the one-loop amplitudes are cancelled by

their counterpart in the real-emission contribution. The latter has been computed by extending the dipole subtraction formalism [6, 9] to the case of massive partons. The infrared contribution that leads to colour correlations proportional to $\boldsymbol{T}_j \cdot \boldsymbol{T}_k$ is produced by soft *and* collinear singularities. The infrared contributions $\Gamma_j^{\rm R.S.}$ are produced by either collinear (but not soft) or soft (but not collinear) singularities.

The singular function $\mathcal{V}_{jk}^{(\rm cc)}$ that controls colour correlations is symmetric with respect to $j \leftrightarrow k$ and depends on the Lorentz invariant $s_{jk} = 2p_j \cdot p_k$ and on the parton masses. In particular, it depends on the the relative velocity v_{jk} of particles j and k:

$$v_{jk} = \sqrt{1 - \frac{m_j^2 m_k^2}{(p_j p_k)^2}} \,. \tag{12}$$

Its explicit expression for non-vanishing masses m_j and m_k is

$$\mathcal{V}_{jk}^{(\rm cc)}(s_{jk}; m_j, m_k; \epsilon) = \frac{1}{2\epsilon}\frac{1}{v_{jk}} \ln\frac{1-v_{jk}}{1+v_{jk}} - \frac{1}{4}\left(\ln^2\frac{m_j^2}{|s_{jk}|} + \ln^2\frac{m_k^2}{|s_{jk}|}\right) - \frac{\pi^2}{6} \,, \tag{13}$$

while for one or two vanishing masses we find

$$\mathcal{V}_{jk}^{(\rm cc)}(s_{jk}; m_j, 0; \epsilon) = \frac{1}{2\epsilon^2} + \frac{1}{2\epsilon}\ln\frac{m_j^2}{|s_{jk}|} - \frac{1}{4}\ln^2\frac{m_j^2}{|s_{jk}|} - \frac{\pi^2}{12} \,, \tag{14}$$

$$\mathcal{V}_{jk}^{(\rm cc)}(s_{jk}; 0, 0; \epsilon) = \frac{1}{\epsilon^2} \,. \tag{15}$$

The functions $\Gamma_j^{\rm R.S.}$ depend on the flavour of the parton j and on the parton masses. In the case of gluons and massless quarks (antiquarks) we have

$$\Gamma_g^{\rm R.S.}(\mu, m_{\{F\}}; \epsilon) = \frac{1}{\epsilon}\,\gamma_g - \tilde{\gamma}_g^{\rm R.S.} - \frac{2}{3}\,T_R \sum_{F=1}^{N_F} \ln\frac{m_F^2}{\mu^2} \,, \tag{16}$$

$$\Gamma_q^{\rm R.S.}(\mu, 0; \epsilon) = \frac{1}{\epsilon}\,\gamma_q - \tilde{\gamma}_q^{\rm R.S.} \,, \tag{17}$$

while for massive quarks (antiquarks) we find

$$\Gamma_q(\mu, m_q; \epsilon) = \boldsymbol{T}_q^2\left(\frac{1}{\epsilon} - \ln\frac{m_q^2}{\mu^2} - 2\right) + \gamma_q \ln\frac{m_q^2}{\mu^2} = C_F\left[\frac{1}{\epsilon} + \frac{1}{2}\ln\frac{m_q^2}{\mu^2} - 2\right] \,. \tag{18}$$

The flavour coefficients γ_j in Eqs. (16) and (17) are

$$\gamma_{j=q,\bar{q}} = \frac{3}{2}\,C_F \,, \qquad\qquad \gamma_g = \frac{11}{6}\,C_A - \frac{2}{3}\,T_R\,N_f \,. \tag{19}$$

The coefficients $\tilde{\gamma}_j^{\rm R.S.}$ parametrize the finite (for $\epsilon \to 0$) contributions related to the RS dependence of the one-loop amplitudes with external massless partons (the massive-quark function in Eq. (18) does not depend on the actual version of dimensional regularization used in the calculation). The transition coefficients $\tilde{\gamma}_j^{\rm R.S.}$ that relate the RS mostly used in one-loop computations were first calculated in Ref. [11]. They are given by

$$\tilde{\gamma}_j^{\rm CDR} = 0 \,, \qquad \tilde{\gamma}_{j=q,\bar{q}}^{\rm DR} = \frac{1}{2}\,C_F \,, \qquad \tilde{\gamma}_{j=g}^{\rm DR} = \frac{1}{6}\,C_A. \tag{20}$$

Note that the dependence on the RS of the ultraviolet and infrared contributions is different. The ultraviolet contribution $\tilde{\beta}_0^{\rm R.S.}$ in Eq. (9) is proportional to the overall power q of $g_{\rm s}$ that controls the amplitude (see Eq. (1)) and it can ultimately be reabsorbed by a process-independent redefinition of the renormalized coupling. The infrared contributions $\tilde{\gamma}_j^{\rm R.S.}$ to Eq. (9) instead depend on the number and flavour of the external massless partons in the amplitude. In the calculation of physical (infrared-finite) quantities this dependence has to be cancelled by computing the corresponding real-emission contributions in a consistent manner, that is, by using the same dimensional-regularization prescription as in one-loop amplitudes [12].

Some comments on the structure of these results, in particular about the ϵ poles and the mass logarithms, are appropriate.

- If there are only massless partons, our result agrees with those in Refs. [4–6]. The structure of the ϵ poles for massive quarks agrees with the results of Ref. [8] and with the QED case considered in Ref. [9].

- The double poles $1/\epsilon^2$ in Eq. (7) are factorized completely and not only in colour space. More precisely, there is a contribution $-\boldsymbol{T}_j^2/\epsilon^2$ from each external *massless* partons j. The simplest way to see that is to expand Eq. (9) in powers of ϵ and then use the colour conservation relation (5), i.e. $\sum_{k\neq j} \boldsymbol{T}_k = -\boldsymbol{T}_j$. One obtains the result

$$\boldsymbol{I}_m(\epsilon, \mu^2; \{p_i, m_i\}) = \sum_{\substack{j \\ m_j=0}} \frac{1}{\epsilon^2} \sum_{k\neq j} \boldsymbol{T}_j \cdot \boldsymbol{T}_k + \mathrm{O}(1/\epsilon) = -\frac{1}{\epsilon^2} \sum_{\substack{j \\ m_j=0}} \boldsymbol{T}_j^2 + \mathrm{O}(1/\epsilon) \tag{21}$$

  that explicitly shows the absence of colour correlations at $\mathrm{O}(1/\epsilon^2)$. Nonetheless, single poles $1/\epsilon$ are both colour- and velocity-correlated.

- The term proportional to $1/\epsilon$ in Eq. (13) is familiar from QED bremsstrahlung, and the imaginary part in Eq. (9) is the corresponding Coulomb phase.

- Since we have

$$\frac{1 - v_{jk}}{1 + v_{jk}} \underset{m_j \to 0}{\widetilde{}} \frac{m_j^2 m_k^2}{s_{jk}^2} \,, \tag{22}$$

  Eqs. (13), (14) and (15) are related by the following formal correspondence

$$\frac{1}{2\epsilon} \ln \frac{m_j^2}{|s_{jk}|} - \frac{1}{4} \ln^2 \frac{m_j^2}{|s_{jk}|} - \frac{\pi^2}{12} + \mathcal{O}\left(\frac{m_j^2}{|s_{jk}|}\right) \longleftrightarrow \frac{1}{2\epsilon^2} \tag{23}$$

  between mass logarithms in the massless limit and $1/\epsilon^2$ poles in the massless theory.

- The various constant (for $\epsilon \to 0$) terms in Eqs. (13)–(18) are important to guarantee the smooth limit in Eq. (8). One can always include additional finite terms in $\boldsymbol{I}$, provided they are smooth in the massless limit. We chose to include the terms that are proportional to $\pi^2 \Theta(s_{jk})$ in Eq. (9).

We also add some other comments on the origin of the last two terms on the right-hand side of Eq. (9). The contributions $\Gamma_j^{\rm R.S.}$ can be related to the Altarelli–Parisi splitting

functions (see Sect. 4). As already mentioned, we have evaluated the colour-correlation term in Eq. (9) by computing the corresponding bremsstrahlung contribution. This directly gives the real part of this term, namely the function $\mathcal{V}_{jk}^{(\mathrm{cc})}$. To obtain the corresponding imaginary part in the square bracket on the right-hand side of Eq. (9), we have exploited the fact that the singular (real and imaginary) part of the colour-correlation term is directly proportional to the the following three-point function:

$$\boldsymbol{T}_j \cdot \boldsymbol{T}_k \int d^d q \, \frac{i}{q^2 + i0} \, \frac{p_j p_k}{[(p_j + q)^2 - m_j^2 + i0] \, [(p_k - q)^2 - m_k^2 + i0]} \; . \tag{24}$$

The contribution of the gluon pole,

$$\frac{i}{q^2 + i0} \to 2\pi \delta_+(q^2) \; , \tag{25}$$

gives $\mathcal{V}_{jk}^{(\mathrm{cc})}$, while the contribution from the poles in the massive propagators,

$$\frac{1}{[(p_j + q)^2 - m_j^2 + i0] \, [(p_k - q)^2 - m_k^2 + i0]} \to -4\pi^2 \delta_+((p_j + q)^2 - m_j^2) \, \delta_+((p_k - q)^2 - m_k^2) \; , \tag{26}$$

gives the corresponding imaginary part (see, e.g., Ref. [15]). Note that in the massive case the imaginary part is more involved than in the massless case, where it is obtained from the real part by the simple analytic continuation $\ln(s_{jk}) \to \ln(-s_{jk} - \mathrm{i}0) = \ln|s_{jk}| - \mathrm{i}\pi\Theta(s_{jk})$ of its overall factor $(\mu^2/s_{jk})^\epsilon$ in Eq. (9).

According to our definition of the insertion operator $\boldsymbol{I}$ in Eq. (7), the finite one-loop contribution $\mathcal{A}_m^{(1)\,\mathrm{fin}}$ still depends on the dimensional-regularization scale μ. This dependence is nonetheless simple, because it is embodied in a contribution proportional to $\ln \mu$. The coefficient of this single-logarithmic contribution is given by

$$\mu^2 \frac{d}{d\mu^2} \mathcal{A}_m^{(1)\,\mathrm{fin}}(\mu^2; \{p_i, m_i\}) = \left(q \, \frac{\beta_0}{2} - \sum_{j=1}^{m} \left[\gamma_j - \frac{2}{3} T_R N_F \delta_{jg} \right] \right) \mathcal{A}_m^{(0)}(\{p_i, m_i\}) \; , \tag{27}$$

Note that μ should not be confused with the renormalization scale. In particular, if renormalized masses (and related Yukawa couplings) do not correspond to the pole-mass definition, but they are $\overline{\mathrm{MS}}$-scheme running masses at the renormalization scale μ_R, the finite contribution $\mathcal{A}_m^{(1)\,\mathrm{fin}}$ can explicitly contain terms of the type $m_i \ln(m_i/\mu_R)$.

3.2 SUSY QCD amplitudes

All the results on QCD amplitudes in Sect. 3.1 can be extended to SUSY QCD processes. A preliminary discussion on the RS and on the small-mass limit is nonetheless necessary.

For a supersymmetric theory CDR (in contrast to DR) is not a consistent RS, because the mismatch between the $d - 2 = 2(1 - \epsilon)$ transverse degrees of freedom of the gluons and the 2 transverse degrees of freedom of the gluinos violates supersymmetric Ward identities. In particular, in the case of on-shell one-loop amplitudes, this leads to violation of the tree-level identity $g_\mathrm{s} = \hat{g}_\mathrm{s}$, between the gluon (gauge) coupling g_s and the $q\tilde{q}\tilde{g}$-Yukawa coupling

$\hat{g}_\mathrm{s}$. Nevertheless, CDR can also be used in SUSY calculations, because supersymmetry can be restored by introducing a proper counterterm [16, 17]. More precisely, we can still use the notation of Eq. (1) (where only g_s appears) in any RS, provided we implement the following RS-dependent relation between the two couplings at one-loop order:

$$\hat{g}_\mathrm{s}^{\mathrm{R.S.}} = g_\mathrm{s} \left[1 + \left(\frac{g_\mathrm{s}}{4\pi} \right)^2 \hat{\gamma}^{\mathrm{R.S.}} \right] \ , \tag{28}$$

where

$$\hat{\gamma}^{\mathrm{DR}} = 0 \ , \qquad \hat{\gamma}^{\mathrm{CDR}} = \frac{2}{3} C_A - \frac{1}{2} C_F \ . \tag{29}$$

As discussed in detail in Sect. 3.1, our factorization formula for one-loop QCD amplitudes smoothly interpolates between the cases of massless and massive quarks. Analogous results, with smooth behaviour with respect to sparticle masses, can be obtained for SUSY QCD. However, in this section we are not going to present such general results because of the following reasons. On one hand, the case of exactly massless SUSY partons has no practical interest. One the other hand, owing to the proliferation of masses in the SUSY particle spectrum, the control of logarithmically-enhanced terms produced by the smallness of the mass of some SUSY partons would require a quite involved presentation and detailed discussions of the various possible cases related to different scenarios of mass hierarchies.

For the sake of simplicity, we thus limit ourselves to considering the case of finite (non-vanishing) squark and gluino masses, $m_{\tilde{q}}$ and $m_{\tilde{g}}$. The factorization formula given below includes *all* the divergent ϵ-poles that appear in this case, and *some* related logarithmic terms.

The one-loop amplitudes for SUSY QCD processes have the same singularity structure as given by Eqs. (7) and (9). The coefficient β_0 in Eq. (9) should simply be replaced by β_0^{SUSY}, the first coefficient of the SUSY QCD beta function:

$$\beta_0^{\mathrm{SUSY}} = \frac{11}{3} C_A - \frac{4}{3} T_R (N_f + N_F) - \frac{2}{3} T_R N_S - \frac{2}{3} C_A \ , \tag{30}$$

where N_S is the number of chiral squark pairs $\tilde{q} = (\tilde{q}_R, \tilde{q}_L)$ ($N_S = N_f + N_F$ in the fully supersymmetric theory). Moreover, the sums over the flavour indices j, k in Eq. (9) run over all parton species ($j, k = q, \bar{q}, g, \tilde{q}, \tilde{g}$). Correspondingly, we have to introduce the flavour functions Γ_j of the external massive gluinos and squarks:

$$\Gamma_j(\mu, m_j; \epsilon) = \boldsymbol{T}_j^2 \left(\frac{1}{\epsilon} - \ln \frac{m_j^2}{\mu^2} - 2 \right) + \gamma_j \ln \frac{m_j^2}{\mu^2} \ , \quad j = \tilde{g}, \tilde{q} \ , \tag{31}$$

where

$$\gamma_{\tilde{g}} = \frac{3}{2} C_A \ , \qquad\qquad \gamma_{\tilde{q}} = 2\, C_F \ . \tag{32}$$

As discussed above, the extension of Eq. (9) to SUSY QCD by using Eqs. (30)–(32) does not include in the insertion operator $\boldsymbol{I}$ all the terms that are logarithmically-enhanced in the limit of small squark and gluino masses. Note however, that the finite contribution $\mathcal{A}_m^{(1)\,\mathrm{fin}}(\mu^2; \{p_i, m_i\})$ of the one-loop SUSY QCD amplitude still fulfils the property

in Eq. (8), provided the massless limit $m_i \to 0$ is restricted to quarks and/or antiquarks ($i = q, \bar{q}$).

The μ-dependence of $\mathcal{A}_m^{(1)\,\mathrm{fin}}(\mu^2; \{p_i, m_i\})$ is still given by Eq. (27), with the replacement $\beta_0 \to \beta_0^{\mathrm{SUSY}}$ and the inclusions of the flavour coefficients $\gamma_{j=\tilde{g},\tilde{q}}$ in Eq. (32).

4 Relation of the singular terms to the Altarelli-Parisi splitting functions

In the case of massless QCD, it is known [4–6] that the flavour functions $\Gamma_j^{\mathrm{R.S.}}$ of Eq. (9) are related to the Altarelli–Parisi splitting functions. In this section we first recall this correspondence and then we sketch how it can be extended to the massive case.

Let us consider, for instance, the collinear splitting of a massless quark. The corresponding Altarelli–Parisi splitting function $\hat{P}_{qq}^{\mathrm{R.S.}}(z;\epsilon)$ in the dimensionally-regularized theory can be written as

$$\hat{P}_{qq}^{\mathrm{R.S.}}(z;\epsilon) = C_F \left[\frac{1+z^2}{1-z} + \frac{h_g^{\mathrm{R.S.}}(\epsilon) - 2}{2}(1-z) \right] \equiv \frac{2C_F}{1-z} + \hat{P}_{qq}^{(\mathrm{reg})\,\mathrm{R.S.}}(z;\epsilon) \, , \tag{33}$$

where $h_g^{\mathrm{R.S.}}(\epsilon)$ is the number of gluon helicity states in the corresponding RS (see Eq. (42)). On the right-hand side we have isolated the contribution that becomes singular in the soft region $z \to 1$ from the remaining regular part $\hat{P}_{qq}^{(\mathrm{reg})\,\mathrm{R.S.}}$.

The soft contribution** has to be combined with the soft terms produced by non-collinear (large-angle) gluon radiation (see Eq. (24)). The terms that are singular both in the soft and collinear regimes lead to the double-pole singularity of the colour-correlation function in Eq. (15). In the massive case kinematics is different from the massless case, and the contribution of these soft terms is embodied in the functions $\mathcal{V}_{jk}^{(\mathrm{cc})}(s_{jk}; m_j, m_k; \epsilon)$, which control the entire colour-correlation part of Eq. (9).

The term $\hat{P}_{qq}^{(\mathrm{reg})\,\mathrm{R.S.}}$ can only lead to collinear singularities (single poles and constants) and it is related to the function $\Gamma_q^{\mathrm{R.S.}}$ in Eq. (17), including its RS dependence [12]:

$$\gamma_q = - \int_0^1 \mathrm{d}z \; \hat{P}_{qq}^{(\mathrm{reg})}(z;\epsilon=0) \, , \tag{34}$$

$$\tilde{\gamma}_q^{\mathrm{R.S.}} - \tilde{\gamma}_q^{\mathrm{CDR}} = \int_0^1 \mathrm{d}z \; \frac{1}{\epsilon} \left[\hat{P}_{qq}^{(\mathrm{reg})\,\mathrm{R.S.}}(z;\epsilon) - \hat{P}_{qq}^{(\mathrm{reg})\,\mathrm{CDR}}(z;\epsilon) \right] \, . \tag{35}$$

Analogous equations relate the other flavour coefficients γ_j, $\tilde{\gamma}_j^{\mathrm{R.S.}}$ in Eqs. (19), (20), (32) to the corresponding terms $\hat{P}_{jk}^{(\mathrm{reg})\,\mathrm{R.S.}}$ of the Altarelli–Parisi splitting functions.

The relation between the functions $\Gamma_j^{\mathrm{R.S.}}$ and the splitting functions can be extended from the massless to the massive case, provided we properly take into account the corresponding *dynamics* and kinematics differences.

**Analogous soft terms, which become singular when $z \to 0$, appear in the Altarelli–Parisi functions of other splitting processes (see the equations in the Appendix).

The dynamics of the splitting processes of massive partons can be described by generalizing the collinear limit to the *quasi-collinear limit*. Let us consider a generic tree-level amplitude $\mathcal{A}^{(0)}_{m+1}(\{p_i, m_i\})$ with $m+1$ external partons. The limit when an internal parent parton (labelled by (jk)) decays quasi-collinearly in two external partons j and k is defined by

$$p_j^\mu \to z p^\mu \quad , \qquad p_k^\mu \to (1-z) p^\mu \quad , \qquad p^2 = m_{(jk)}^2 \quad , \tag{36}$$

with the constraint

$$p_j \cdot p_k, \; m_j, \; m_k, \; m_{(jk)} \to 0 \qquad \text{at fixed ratios} \qquad \frac{m_j^2}{p_j \cdot p_k}, \; \frac{m_k^2}{p_j \cdot p_k}, \; \frac{m_{(jk)}^2}{p_j \cdot p_k} \; . \tag{37}$$

The quasi-collinear limit obviously differs from the collinear limit because the splitting partons are massive. However, the key difference between the two limits is given by the constraint that the on-shell masses squared have to be kept of the same order as the invariant mass $(p_j + p_k)^2$, while the latter become small.

It can be shown that in the quasi-collinear limit the tree-level squared amplitude fulfils a factorization formula similar to the analogous formula for the collinear limit. We have

$$|\mathcal{A}^{(0)}_{m+1}|^2 \;\sim\; |\mathcal{A}^{(0)}_{m}|^2 \, \frac{2\mu^{2\epsilon} g_s^2}{(p_j+p_k)^2 - m_{(jk)}^2} \; \hat{P}^{\rm R.S.}_{(jk),j}(z;\epsilon;\{\mu_l^2\}) \; , \tag{38}$$

where the m-parton matrix element on the right-hand side is obtained from $\mathcal{A}^{(0)}_{m+1}$ by replacing the partons j and k with the single parent parton of momentum p. The function $\hat{P}^{\rm R.S.}_{(jk),j}(z;\epsilon;\{\mu_l^2\})$ generalizes the customary d-dimensional Altarelli–Parisi splitting function to the case of quasi-collinear splitting. It has the usual dependence on ϵ and on the longitutinal-momentum fraction z, plus an additional dependence on the parton masses indicated by the variables $\{\mu_l\}$:

$$\mu_l^2 = \frac{m_l^2}{(p_j+p_k)^2 - m_{(jk)}^2} \quad , \tag{39}$$

where (in general) m_l^2 stands for any quadratic combination of the masses of the partons involved in the splitting process ($m_l^2 = m_j^2, m_k^2, m_{(jk)}^2, m_j m_k, \dots$ and so forth).

For instance, in the case of the quasi-collinear splitting $q \to q + g$ of a massive quark ($p_q^2 = m_q^2, \; p_g^2 = 0$), the analogue of the massless splitting function in Eq. (33) is (see Eq. (43) and Refs. [8,9]):

$$\hat{P}^{\rm R.S.}_{qq}\left(z;\epsilon;\frac{m_q^2}{2p_q p_g}\right) = \frac{2C_F}{1-z} + \left[\hat{P}^{(\rm reg)\,R.S.}_{qq}(z;\epsilon) - C_F \frac{m_q^2}{p_q p_g}\right] \; . \tag{40}$$

The 'regular' part of the splitting function is now given by the contribution in the square bracket. It contains a term that explicitly depends on m_q, in addition to the regular part that appears in the massless case (see Eq. (33)).

Comparing the regular parts of the splitting functions in Eqs. (33) and (40), we can understand the difference between the the massless and massive functions Γ_q in Eqs. (17) and (18). This difference amounts to the correspondence

$$\frac{1}{\epsilon}\left(\gamma_q - \epsilon \tilde{\gamma}_q^{\rm R.S.}\right) \longleftrightarrow \gamma_q \ln \frac{m_q^2}{\mu^2} + C_F \frac{1}{\epsilon}\left(1 - \epsilon \ln \frac{m_q^2}{\mu^2} - 2\epsilon\right) \; . \tag{41}$$

In both the massless and massive cases, γ_q is given by Eq. (34). The coefficient in front of γ_q is obtained by performing the integration of the propagator factor $1/p_q p_g$ (see Eq. (38)) over the relative angle θ_{qg} between the quark and the gluon. In the massless case this integration is singular when $\theta_{qg} \to 0$ and it leads to the single pole $1/\epsilon$ on the left-hand side. In the massive case, the integration is kinematically regularized at a cutoff value $\theta_{qg} \gtrsim m_q/E_q$ and it leads to the logarithmic behaviour of the first term on the right-hand side. The contribution inside the round bracket on the right-hand side is instead produced by the term of Eq. (40) that explicitly depends on m_q. Here, the angular integration, which is dominated by the region $\theta_{qg} \lesssim m_q/E_q$, produces a constant term, while the integration over the gluon energy $E_g \sim (1-z)E_q$ is divergent in the soft region $z \to 1$ and it leads to the single pole $1/\epsilon$ in front of the round bracket.

The generalized Altarelli–Parisi functions for the quasi-collinear limit of the other splitting processes in QCD and SUSY QCD are listed in the Appendix. The expressions of the flavour functions in Eq. (16) and Eq. (31) are related to the splitting functions in Eqs. (44), (45) and Eqs. (46), (48), respectively.

5 Summary

In this paper we have discussed the singular behaviour of on-shell QCD and SUSY QCD amplitudes at one-loop order in the presence of massive particles. The complete structure of the ultraviolet and infrared singularities is described by the colour-space factorization formula given in Sect. 3. The factorization formula is universal, i.e. valid for any amplitude, and the singular factors only depend on the flavours and momenta of the coloured external legs. Moreover, the factorization formula is given in such a form that the corresponding formula for massless QCD partons is smoothly recovered by simply letting the masses approach to zero.

Our factorization formula can be useful both to check explicit evaluations of one-loop amplitudes and to organize their calculations in terms of divergent parts and finite remainders. Furthermore, in the asymptotic regime where the parton masses are much smaller than any of the relevant kinematic invariants, the formula can also be used to directly obtain (apart from vanishing corrections when the masses tend to zero) the one-loop massive amplitude from the corresponding massless amplitude, without explicitly computing the former. In the general context of NLO calculations of jet observables, our one-loop results are useful for setting up the integration of tree-level amplitudes in such a way as to construct process-independent techniques for infrared cancellations.

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Appendix: Quasi-collinear dynamics and generalized Altarelli–Parisi splitting functions

In this appendix we list Altarelli–Parisi splitting functions in QCD and SUSY QCD, supplemented by the mass terms that are relevant in the quasi-collinear limit discussed in Sect. 4 (see Eqs. (36)-(39)). In the expressions given below all the mass terms can be parametrized by the variable $\mu_{jk}^2 = (m_j^2 + m_k^2)/[(p_j + p_k)^2 - m_{(jk)}^2]$. We include the dependence on the RS, which is parametrized by the corresponding number $h_g^{\rm R.S.}(\epsilon)$ of gluon polarizations,

$$h_g^{\rm CDR} = d - 2 = 2 - 2\epsilon \,, \qquad h_g^{\rm DR} = 2 \,. \tag{42}$$

Note that our expressions refer to Altarelli–Parisi splitting functions after average over the azimuthal angle identified by the collinear direction. The azimuthally averaged functions are relevant to discuss the RS dependence [12] and, in general, they do not coincide with the splitting functions averaged over the polarizations of the parent parton. Since we are considering the splitting process $(jk) \to j\,k$, the splitting functions fulfil the obvious symmetry relation $\hat{P}^{\rm R.S.}_{(jk)k}(z;\epsilon;\mu_{kj}^2) = \hat{P}^{\rm R.S.}_{(jk)j}(1-z;\epsilon;\mu_{jk}^2)$.

Quarks and gluons:

$$\hat{P}^{\rm R.S.}_{qq}(z;\epsilon;\mu_{qg}^2) = C_F \left[\frac{2z}{1-z} + \frac{1}{2}\, h_g^{\rm R.S.}(1-z) - 2\mu_{qg}^2\right] , \tag{43}$$

$$\hat{P}^{\rm R.S.}_{gq}(z;\epsilon;\mu_{q\bar{q}}^2) = T_R \left[1 - \frac{2}{d-2}\left(2z(1-z) - \mu_{q\bar{q}}^2\right)\right] , \tag{44}$$

$$\hat{P}^{\rm R.S.}_{gg}(z;\epsilon) = 2C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + \frac{h_g^{\rm R.S.}}{d-2} z(1-z)\right] . \tag{45}$$

Gluinos and gluons:

$$\hat{P}^{\rm R.S.}_{\tilde{g}\tilde{g}}(z;\epsilon;\mu_{\tilde{g}g}^2) = C_A \left[\frac{2z}{1-z} + \frac{1}{2}\, h_g^{\rm R.S.}(1-z) - 2\mu_{\tilde{g}g}^2\right] , \tag{46}$$

$$\hat{P}^{\rm R.S.}_{g\tilde{g}}(z;\epsilon;\mu_{\tilde{g}\tilde{g}}^2) = C_A \left[1 - \frac{2}{d-2}\left(2z(1-z) - \mu_{\tilde{g}\tilde{g}}^2\right)\right] . \tag{47}$$

Squarks and gluons:

$$\hat{P}^{\rm R.S.}_{\tilde{q}\tilde{q}}(z;\epsilon;\mu_{\tilde{q}g}^2) = C_F \left[\frac{2z}{1-z} - 2\mu_{\tilde{q}g}^2\right] , \tag{48}$$

$$\hat{P}^{\rm R.S.}_{g\tilde{q}}(z;\epsilon;\mu_{\tilde{q}\bar{\tilde{q}}}^2) = T_R \,\frac{1}{d-2}\left[2z(1-z) - \mu_{\tilde{q}\bar{\tilde{q}}}^2\right] . \tag{49}$$

Equations (48)–(49) are valid for the superpartners of both the left- and the right-chirality quarks.

Neglecting the mass terms, which are proportional to μ_{jk}^2, the splitting functions coincide with those reported in Ref. [17] for the CDR scheme. We do not consider the splitting functions produced by the Yukawa coupling $q\tilde{g}\tilde{q}$, because, as long as the gluino and squark masses are finite, they do not produce any singular terms when $\epsilon \to 0$.

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